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ABSTRACT

The author reviews five cost-effectiveness basic models including log-log correlational, general utility theory, simultaneous equations, nonlinear theoretical, and feedback. Several suggestions are made to improve the models and increase the domain of problems that can be considered by the models. In the second part of the paper, the author presents a refined feedback model that can describe the relationship among inputs such as students, resources, parent and community factors, and outputs such as achievement. Also included are a ranked sensitivity technique for indicating research priorities and an educational cost-effectiveness bibliography. (Author/RA)

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A CRITIQUE OF SELECTED REFERENCES ON EDUCATIONAL COST EFFECTIVENESS WITH A RESOURCE - UTILITY MODEL TO IMPROVE RESOURCE ALLOCATION

EA 004 016

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June 8, 1971
Master's Project
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SUMMARY

The author reviews five basic models: a log-log correlational, general utility theory, simultaneous equations, non-linear theoretical, and feedback. Of these models, the first and third are derived from data solely, the second is a hypothetical model to determine the utility of programs, and the last two are developed from intuition or elementary theory. Several suggestions are made to improve the models and increase the domain of problems that can be considered by the models.

In the second part of the paper, a model based on the models reviewed and other research is presented using the feedback model as a basis. The author, in more detail, describes the relationship between inputs of students, resources, parent and community factors, and outputs such as achievement. Additionally the author presents the ranked sensitivity technique for indicating which areas should have a higher priority in research. An extensive bibliography of educational cost effectiveness articles is included.

PURPOSE

This paper is an effort to elicit interest in operations research in education.

Starting with the presentation of certain of the techniques currently available to education, the author intends to inform the reader of what is being done in the area of educational cost effectiveness. Expanding certain techniques to include a larger class of problems, the author hopes to simulate discussion about educational cost effectiveness uses. By creating interest among other operations researchers, the author attempt to increase the flow of information about techniques not currently used in education but that may be applicable. This paper is a vehicle for such discussions, and by being somewhat controversial in nature, will generate the intended interest.

INTRODUCTION

Today there is much concern about the taxpayer's revolt in education. In all school districts throughout the United States, taxpayers are closely examining the requests for school funds. Over the last 10 years the expenditure per student in elementary and secondary schools has increased by more than 100%.

Presently operations research has had little application in elementary and secondary education. In a limited number of districts class scheduling, bus routing, meal planning, and school location is done using scheduling theory or linear programming. Few, if any, districts have tried to optimize the total school expenditure. Several reasons exist for this. Among them are lack of: a well defined objective function, a relationship of inputs to outputs, specific costs of each program in a school. Since 1965 several states, Nevada, Texas, California, Washington, have implemented Program Budgeting as a technique for planning future requirements and as a method of accounting. This method is satisfactory for planning next years expenditures by program because it lists all past expenditures; however, the decision maker receives little insight into which programs should be implemented.

A cost effectiveness approach is one method to determine which program should be implemented. Two general methods of cost effectiveness are resource allocation among projects and components of the projects, and optimization of the use of

resources once the resources have been allocated. Using a model to determine whether project A or project B should be funded is an example of the first method while using linear programming to optimize bus routes is an example of the second method. Both methods could be quite useful.

In this paper the author will review some specific allocations techniques, commenting on the applicability of each technique. Of the five basic models reviewed, log-log correlational, general utility theory, simultaneous equation, non-linear theoretical and feedback, only one, the non-linear theoretical model, actually explains the allocation procedure. While the others do not actually explain how to allocate resources, they do determine which programs or resources contribute the most to output of a model. From this an allocation procedure can be constructed, although such a procedure may not be easy to implement. As mentioned in the summary, the author constructs a model utilizing various parts of specific models reviewed and original work about school cost incurrance done by the author and Ronald Randall, an educational consultant. The basic structure is a feedback loop approach used by T. Jones in the paper reviewed later. One reason why this approach was selected was that current decisions are not necessarily based on the present status of a system because of possible delays in the flow of information.

In the bibliography are papers describing resource allocation. Several bibliographies containing both resource allocation and optimization of the use of resources are presented.

CURRENT WORK IN EDUCATIONAL COST EFFECTIVENESS

In the last 10 years, many studies in public school education have been conducted concerning the cost effectiveness of various resources. Several of these studies are reviewed in this paper. Two categories, multiple correlations studies and longitudinal studies, contain most of the studies completed in educational cost effectiveness.[1]

By using a multiple correlation study, a researcher attempts to relate differences in achievement to differences in student, teacher, school and community characteristics. Usually this is done by linear regression using data sampled from one instant in time. Associated types of studies include partitioning of variance and factor analysis.

A longitudinal study collects information about the experimental subjects over several time intervals and from this data attempts to construct input-output relationships.

Eric Hanushek (10) derived two log-log statistical relationships between certain input and output variables. Average data from 713 schools was used to construct one model for whites and one for Negroes. Because of the effect of unmeasured variables, the results are subject to question.

Hanushek propose the following model:

(1) $$A_{it} = f(\beta_i^{(t)}, P_i^{(t)}, I_i, S_i^{(t)})$$

A_{it} = vector of educational outputs of the ith student at time t

$\beta_i^{(t)}$ = vector of family inputs to education of the ith student cumulative to time t

$P_i^{(t)}$ = vector of peer influences of the ith student cumulative to time t

I_i = vector of innate endowments of the ith student

$S_i^{(t)}$ = vector of school inputs to ith student cumulative to time t.

Using sixth grade white students in 471 schools and sixth grade Negro students in 242 schools, Hanushek derived two log - log regression relationships between input variables and student achievement, as measured by student verbal test scores, that proved to be superior to linear relationships. The coefficients estimated from the sample data can be interpreted as elasticities because of the model's multiplicative form.

Using this kind of analysis presents several difficulties. A correlational study does not prove cause-and-effect relationships, but merely that at a certain instant in time a large value of the input variable is associated with a high value of student achievement, assuming the correlation is positive. For example, if the sample average value of the input variable of interest alternated between -1 and +1 every month and the student achievement were +1 irrespective of the input variable, then the first month the correlation would be -1.0, and the second month it would be +1.0. Rather than calculate the correlations and then determine which variables are significantly

correlated, a better procedure would have been to hypothesize that certain variables were correlated, set a level of acceptance on the correlation and then calculate the correlations. The level of acceptance is based on the number of sample points and the judgement of the person making the hypothesis. This reduces the chance of accepting a spurious correlation.

Results from such a study are not useful for decision making unless one can show that there exists a cause-effect relationship between the input value and the student achievement. The input variables in a particular survey may not contribute to achievement but are proxies for unmeasured variables that do contribute.[2] Positive correlations of these proxy variables with student achievement would lead a decision maker to believe that an increase in the proxy variable would increase achievement when in reality, the increase in input may have little or no effect. Kiesling's (14) study indicates that there is a correlation between additional expenditures and increased achievement. Yet the decision maker is not guaranteed that by increasing expenditures, achievement will increase, but that if he spends this additional money to purchase a specific mix of resources, that the specific mix will tend to increase achievement.

The usefulness of correlations can be increased in certain cases. If on several independent studies, the same input variables have high correlations, one can then test statistically whether that particular variable, or closely related ones, has an effect that is not spurious. To some extent if

one survey is large enough, the above is true; however, spurious correlations may exist because the survey was taken at an instant in time. In particular taking a survey during the first week of school could lead to results not applicable to other times in the year. One must remember though, that just because a specific variable had a significant correlation with output at a particular instant and not significant correlation at later time, does not mean the correlation is spurious; the model could have parameters that change over time. Unfortunately, the Coleman Report was taken for just one instant of time, so one cannot test the relationships at a later time unless other data exists.

Hanushek did not include data from other times in his paper.

Another disadvantage of Hanushek's procedure is that of noise, those variables affecting achievement but not measured, in the system at the particular time the data survey was taken. Whether this effect of noise is large is difficult to determine because a theoretical model relating inputs to achievement is almost non-existent.

A learning theory developed by James Bright suggested that a measure of efficiency is log-log related to the cumulative number of repetitions of a given task; however, this model is for a production line process for a group and not individuals. Additional research into the applicability in public education of Bright's theory would enhance the quality of the paper.

One additional difficulty is with the data collected. According to Hanushek, most data surveys contain only one measure of output, and this leads to a difficulty in determining the utility of various alternative educational procedures. Several surveys can be used to obtain several measures of output, but combining the information of these surveys presents a major problem. The surveys do not have an identical set of input conditions so that the output measures are not necessarily derived from comparable input vectors. With so little theory to support a micro-analysis model, adjustments for differences in the input vectors are made from judgements or correlational studies, not a very reliable procedure.

Using the Equality of Educational Opportunity Survey (Coleman Report), Hanushek compares aggregated data from 713 school in the urban centers in the Northeast. By using this aggregated data, the model proposed was modified to use school averages instead of individual student differences. This procedure assumes that the average utility and achievement, e.g. $\underline{A}_t = f(\underline{B}^t, \underline{P}^t, \underline{I}^t, \underline{S}^t)$ where the underscore indicates average value, are independent of the individual student characteristics. There is reason to believe that this assumption is false. For example, if f were linear in P, I, S, but log-log related in B the $\underline{A}_t \neq f(\underline{B}^t, \underline{P}^t, \underline{I}^t, \underline{S}^t)$. Furthermore, a decision maker may place a much higher utility on a unit of achievement by a student who is below average than by one who is above.

Hanushek's work indicates the general level of research being conducted in education. The absence of a theoretical

micro-interaction model hinders the development of meaningful decision making.

J. P. Amor and J. S. Dyer (1) present a model for converting test scores, although other output measures could be used, into utility. By computing the utility per dollar for each project, one can optimize the total expenditure. If project costs are incurred in discrete units, this procedure may not yield an optimal solution. Amor and Dyer developed a model utilizing the following functional form:

$$(2) \qquad f(a_1, a_2, \ldots a_n) = \sum_{i=1}^{n} w_i f_i(a_i)$$

where a_i is the percentile score on a device measuring achievement in a specific goal area,

f_i is a function which converts test scores into the decision maker's utility units,

w_i is a weighting factor indicating the relative importance of each of the goals.

To make decisions the criterion of expected change in utility per dollar is used.

$$(3) \qquad \frac{E(\Delta U_i)}{c_j} \approx \frac{w_i\left[\sum_{K=1} f_i(\Delta_K) - f_i(a_i^0)\ P_i(\Delta_K | a_{ij})\right]}{c_j}$$

where $E(\Delta U_i)$ is the expected change in utility in area i,

c_j is the cost of program j,

w_i is the relative importance to the decision maker of area i,

$f_i(\Delta_K)-f_i(a_i^0)$ is the change in utility associated with increases in percentile scores,

$P_i(\Delta_K|a_{ij}^0)$ is the probability of a specific change Δ_K in utility in area i if program j is used and the previous percentile scores for area i.

By computing $E(\Delta U_i)/c_j$ for all programs considered, the program with the greatest increase in utility per dollar can be selected.

Ignoring the facts that the function form f_i and the achievement level a_i may not be known accurately, the procedure still has serious deficiencies. The criterion of using the maximal increase in utility per dollar is not satisfactory if project costs occur in non-linear forms, especially in descrete quantities. Amor and Dyer do not consider a budgetary constraint in the decision procedure. For example, if a project has a cost of \$10,000 and $E(\Delta U_i)/\$10,000 = .00004$ was the greatest value of $E(\Delta U_i)/c_j$, Amor and Dyer would choose that project. Yet, if only \$8,000 were available, such a procedure would lead to an incorrect solution. In connection with this, $E(\Delta U_i)/c_j$ may not be constant, i.e. as total cost increases by R%, Utility increases by S%, as is the case in a project with fixed administrative costs and a specific variable cost per student,so that each level of implemention would have to be considered another project. A decision maker may consider an increase in achievement by a particular segment of the student population to be associated with a large increase in utility. In such an event each increment in achievement could have an associated utility and the total change in utility could be

computed by weighting the individual utilities by the number of people accomplishing a particular increase in achievement, but Amor's model permits only average increase in achievement for the whole population to be considered.

This objection is easily overcome by redefining goal areas as goal areas for subpopulation ℓ, $\ell = 1,2, \ldots m$, e.g. a_i is replaced by $(a_{1i}, a_{2i}, \ldots a_{mi})$ where the subscript i is replaced by the subscripts ℓi. Similar changes are made for f_i and w_i. The model then becomes

$$f(a_{ij} \cdots a_{\ell i} \cdots a_{mn}) = \sum_{\ell=1}^{m} \sum_{i=1}^{n} w_{\ell i} f_{\ell i}(a_{\ell i}).$$

The above method may not be satisfactory either, for the decision maker may place a high utility on how many students have a particular increase in achievement. Utility is then a function of the sum of the individual utilities, which are a function of the number of students and a particular increase in achievement.

The decision procedure could be revised to use dynamic programming to determine the allocation of dollars to projects that yielded the greatest increase in associated utility.

Let N equal the number of potential projects, d_j, $j = 1, 2, \ldots, N$ be the amount of money to be spent on each project subject to the constraint

$$\sum_{i=1}^{N} d_j \leq M$$

where M is the total amount of money available. Return $R_j(d_j)$ is the utility associated with project j. For this problem the objective function would be to maximize

$$\sum_{j=1}^{N} R_j(d_j).$$

The state variable x_j is the amount of money available for allocation to projects 1, 2, ..., j, i.e. $x_{j-1} = x_j - d_j$. Note that the decision variables d_j can be transformed into decision vectors by adding decision variables as to how many students of each subpopulation will be in project j.

Finally, the Amor and Dyer model overlooks the possibility of cross-goal effects of a project. If the goal area is reading, an increase in achievement in that area could affect other areas such as history, science, current events, where the ability to read is an essential skill. By summing the right hand of (3) over the i goal areas, the total change in utility per dollar can be computed provided f is a separable function.

If f is not separable, the right hand side of equation (2) is identical to the left hand side. This increases the difficulty in the implementation of the model because the decision maker must be able to define $f(a_1,a_2, \ldots, a_n)$ instead of the $f(a_i)$ $i = 1, 2, \ldots, n$. The function $f(a_1,a_2, \ldots, a_n)$ could be separable but not into n terms. If this is the case then the implementation of the model is less difficulty than the non-separable case but more difficult than the original model because the difficulty in determining common utility in different goal areas. These objections are not important if one can find an n-term separable function to represent one's utility.

In Amor and Dyer's paper one assumes that the total utility of the decision maker is the weighted sum of the goal utilities, yet the decision maker's units of utility for each goal area may be different and perhaps not convertible from one to another. Trying to compare the utility of one unit of increase in social values with that of one unit of increase in reading is a major difficulty. Unfortunately, the construction of such a function that converts the n-tuple of utility measures into a common measure is not easy.

For severely restricted cases as outlined above, the model is inadequate; however, the use of this model in more general situations will not result in a correct analysis either. Because the sensitivity of determining the correct solution is not known, one cannot determine if the unaltered model will yield the optimal or new optimal decision.

Levin's (18) "A New Model of School Effectiveness" is a source of much insight into the problem of modeling the educational process. One major assumption made in educational experiments according to Levin is ceteris paribus, i.e. that all things not measured in the experiment are equal. Yet this assumption by educators is not justified because the experiments conducted in education do not lead to conclusive results, not even always to consistent results. Either the experiements are poorly conceived and operated, or noise variables contribute significantly to the output measures of achievement.

Because of the lack of theory describing the educational process and not having a common measure for the multiple goals

of education, one cannot specify the relationships between specific inputs and outputs.[3]

One idea that the previous studies neglected is that observations of the educational system are not the most efficient or the maximum obtainable with the given inputs. This is similar to observing the horsepower of a poorly tuned engine; an engineer would have great difficulty in determining the combustion equations. Unfortunately the development of the education production function is hampered by three additional problem areas: lack of knowledge of the relevant production set, lack of discretion in the use of inputs, and lack of incentive to improve the educational process.[4]

Levin has developed a simultaneous equation system that has multiple output measures instead of just one measure. By solving the set of equations with a two stage least squares procedure, interaction of the output variables was permitted. One disadvantage of this procedure is that it does not recognize time delay effects of the output variables on other variables in the model.

Using Hanushek's model as a beginning, Levin expands the list of arguments to include:

$A_{2_{it}}$ = a measure of the student's sense of efficacy or fate control at t,

$A_{3_{it}}$ = a measure of educational motivation of the ith student at t,

$A_{4_{it}}$ = parents' educational expectation for the student at t.

The complete model is the following:

(4) $A_{1_{it}} = g(F_{i(t)}, S_{i(t)}, P_{i(t)}, O_{i(t)}, I_{i(t)}, A_{2_{it}}, A_{3_{it}}, A_4)$

where $F_{i(t)}$, $S_{i(t)}$, $P_{i(t)}$, $O_{i(t)}$, $I_{i(t)}$, are as Hanushek defined them. Vectors A_{2it}, A_{3it}, A_{4it} are unique in that not only is each assumed to affect achievement, but also each is affected by the level of achievement. Further assumptions were made that particular variables contributed to changes in only one of the four A variables, A_{1it}, A_{2it}, A_{3it}, A_{4it}, resulting in a four-equation system:

(5) $A_{1it} = g_1(F_{1i(t)}, S_{1i(t)}, P_{1i(t)}, O_{1i(t)}, I_{1i(t)}, A_{2it}, A_{3it}, A_{4it})$

(6) $A_{2it} = g_2(F_{2i(t)}, S_{2i(t)}, P_{2i(t)}, O_{2i(t)}, I_{2i(t)} A_{1it}, A_{3it}, A_{4it})$

(7) $A_{3it} = g_3(F_{3i(t)}, S_{3i(t)}, P_{3i(t)}, O_{3i(t)}, I_{3i(t)}, A_{1it}, A_{2it}, A_{4it})$

(8) $A_{4it} = g_4(F_{4i(t)}, S_{4i(t)}, P_{4i(t)}, O_{4i(t)}, I_{4i(t)}, A_{1it}, A_{2it}, A_{3it})$

If one assumes that the g_1 are linear equations as Levin does in his solved example, then the four equation problem can be solved using standard linear algebra techniques.

Coefficients are determined three ways: by ordinary least squares, two stage least squares, and algebraic substitution all using data from the Coleman Report. The ordinary least squares procedure estimates the coefficients of each of the equations (5), (6), (7), (8) independently of any results of the other three equations. Two stage least squares procedure estimates the coefficients of all equations simultaneously. This may yield a different set of coefficients because of the possible interaction effects of the variables. If the output data A_{jit} did not depend on the other output information A_{kit} $k \neq j$, then the ordinary least squares and two stage least squares would yield coefficients for each variable that would be approximately equal. But the observation A_{jit} cannot be divided into two observations, the one depending on equation $(j + 4)$ and one on the other three equations, because the surveys cannot explicityly determine the division of the observation. Therefore, statistical methods are used to determine the division of the A_{jit}. By using three equations instead of four, a different set of coefficients will result. Levin solved equations (5), (6), (7) by two stage least squares and then substituted the resulting values of A_{1it}, A_{2it} and A_{3it} into equation (8) which was solved by ordinary least squares. No reason was stated for this change.

To reduce the number of variables in the model, Levin eliminated those input variables that had little or no statistical effect on achievement output. In the particular study, the innate characteristics vector was deleted due to lack of data.

While the general caliber of this study is above the previous ones, several theoretical and statistical deficiencies exist. As described in Hanushek's paper, a linear equation may not fit the data as well as some non-linear forms. In such a case, one does not have a general method to solve the system of equations and guarantee that a solution or lack of a solution can be demonstrated. Many techniques exist for solving special kinds of non-linear equations such as transformation to linear equations and piecewise linear approximation. More importantly, if one assumes that A_{1it}, A_{2it}, ..., A_{4it} interact with each other, there is no reason to believe that the current value of each of these variables or the sum of all previous values before t is the deciding value. For example, current levels of achievement may not be known to parents, students, and the community. To cite another example, the parents' educational expectation, A_{4it}, as Levin states, depends on the past performance level of the students, not necessarily on the sum of all previous performance to time t.[5]

Community influences, O_{lit}, may be based on information several time periods old, e.g., "What's good enough for me is good enough for today's student." Additionally, governmental influences may be based on studies or surveys that took a number of time periods to complete and analyze.

Each of the $F_{ji(t)}$, $S_{ji(t)}$, $P_{ji(t)}$ and $O_{ji(t)}$ are controllable variables to some extent. If Levin's model is assumed to be a good representation of reality, then one can optimize

expenditures by applying non-linear programming. The objective function would be to maximize

$$U = \sum_{j=1}^{4} \sum_{i=1}^{N} f_{ij}(A_{jit}) \cdot$$

Where f_{ij} is a function which transforms output into utility, such as in Amor and Dyer's model. There are N variables in the input vectors. Equations (5), (6), (7), (8) are constraints along with equations describing the cost of changing the input variables and one equation that represents the budgetary constraint. Nonnegativity restrictions are added where necessary; so are other restrictions on the solution space, such as state laws.

Levin states that his interpretations are highly speculative because of the reasons discussed in the beginning of this section. One significant interpretation that should be examined is that if the ordinary least squares estimate of the coefficient varies greatly from the simultaneous equation estimate, then that particular variable has indirect effects that constitute the difference. If all variables had a zero time lag, this would be a reasonable assumption; however, with time delays, the vector inputs measured may not represent the true state of the educational system or the state used to determine interaction effects. The apparent indirect effects could be caused by previous values of variable in the particular equation examined.

Roger Sisson (27,28)of the University of Pennsylvania Management Science Center has developed two models relating resource inputs to educational outcomes. The first model uses

a static exponential function or a non-linear model derived from communication theory developed by Martin Stankard at the University of Pennsylvania. The second follows the students through the public schools and uses recursive relationships to predict enrollment.

Prefacing the first model, Sisson states three reasons why operations research has not made a major contribution to education. Lack of money and a learning theory have hindered model development and this led educators to conduct experiments which yielded inconclusive results, because of the large amount of noise in the experiments. If a comprehensive learning theory existed, then the noise variables could be isolated and their effects experiementally or statistically reduced so that the validity of the theoretical model could be tested.

For the static model, Sisson has developed two editions, the earlier one based on communication theory and the latter one based on a power function. Describing the first model is the following equation:

$$(9)\quad Z = \left\{\left[f_1(S) + K_1\left(\frac{M}{1+S}\right)\right](1-e^{-A/A_o}) + f_2\left(\frac{C}{1+P}\right)\right\} f_3(P)$$

Z = achievement (standardized test scores),

S = staff per student,

M = material per student in dollars,

A = space per student in square feet,

C = community relations effort in dollars per student,

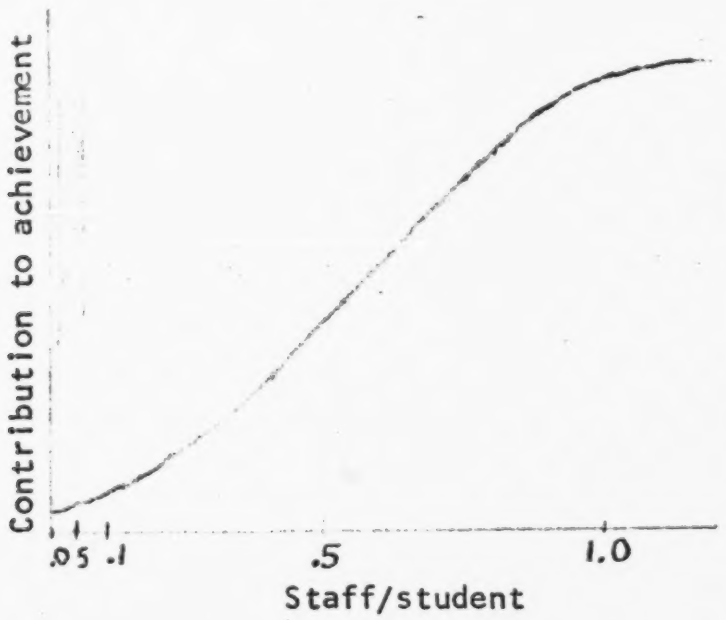
P = average level of parental education,

K_1, A_o ; constants

f_1 = logistic function; $f_1(X) = \frac{K_i}{1 + \exp(K_{i+1} + K_{i+2}X)}$

The first term, $f\ (S) + K_1(\frac{M}{1+S})$, relates the staff ratio to achievement. Suggested by the first term is an "S" curve shaped similar to the learning curve developed by James Bright.[6]

Diagram 1



From the diagram one notes that the more staff per student, the greater the achievement, up to a ratio of .8 or .9 where diminishing return effects are noticed. Material expenditure has a positive effect on achievement and also an interaction effect with the amount of staff per student, e.g. a high staff ratio can offset some or all of the effects of a low material budget. Crowding of schools has a one sided negative effect. If A is greater than A_o, then adequate space for learning is available and added space makes little contribution to additional achievement. Parental effects have a strong influence on the achievement output because in the model, parental

average education modifies the entire equation and the community effects. Similar to the material term, the community term is modified by the educational level in the community.

This model was one of the first attempts to relate the achievement output to several inputs on a basis other than regression analysis. For such a model the deficiencies are serious but not unexpected. Undoubtedly the communication process is much more complicated than illustrated by this model. Several questions can be asked about the validity of the input variable as measures of their representative areas. Rather than critique this first edition, the author will present the second edition with comments, some of which will apply to both editions of the model.

In the second edition of the resource achievement model, the following equation is used:

$$(10) \qquad D = \sum_i A_i n_i (\xi + T_i)^{\mu_i} (\psi + S_i)^{\nu_i} (\phi + M_i)^{N_i}$$

where $A_i, \mu_i, \nu_i, N_i, \xi, \psi, \phi$ are constants

D = achievement measured by tests

n_i = number of students in grade i

T_i = ratio of teachers to students in grade i

S_i = space per student in grade i

M_i = dollars spent on material per student in grade i

$i = 1, 2, \ldots, N$

and is subject to a constraint

$$\sum_i [C_{1i}(n_i T_i) + C_{2i}(n_i S_i) + M_i n_i] \leq B$$

where C_{1i} = salary of a teacher in grade i

C_{2i} = expenditure per square foot in grade i

B = budget available

Proceeding by the LaGrange multiplier method, the constrained optimization problem has 3N + 1 simultaneous equations and variables. To maximize D, the following must be true:

$$\frac{C_{1i}}{M_i}(\xi + T_i) = \frac{C_{2i}}{\nu_i}(\psi + S_i) = \frac{1}{N_i}(\phi + M_i)$$

$$= \left[\frac{A_i}{\lambda} (\xi + T_i)^{\mu_i}(\psi + S_i) (\phi + M_i)^{N_i}\right]$$

Sisson solves the particular case where $\xi = \psi = \phi = 0$ which yields:

$$T_i = \frac{\mu_i}{N_i C_{1i}} M_i \qquad S_i = \frac{\nu_i}{N_i C_{2i}} M_i$$

$$M_i = \frac{A_i N_i}{\lambda} n_i T_i^{\nu_i} S_i^{\mu_i} M_i^{N_i} \qquad (i = 1, \ldots N)$$

Two examples are used to illustrate the model; the second appears below.

N = 2 C_{11} = 8500 C_{21} = 3 μ_1 = .9 ν_1 = .60 n_1 = .09

C_{12} = 8800 C_{22} = 3 μ_2 = .85 ν_2 = .57 n_2 = .08

$A_1 \equiv A_2 \equiv A$ B = 617,366

500 = number of students entering grade 1

$\beta_i = k_i T_i^{I_i} S_i^{N_i} M_i$ σ_i = % of students promoted from grade i

$\alpha_i = 1 - \beta_i$ = % of students not promoted from grade i

$n_i = K/\beta$

$I_1 = .05$ $m_1 = .04$ $\sigma_1 = .004$ $K = 400$ $k_1 = .85$

$I_2 = .06$ $m_2 = .05$ $\sigma_2 = .005$ $k_2 = .76$

By solving the necessary conditions for maximum achievement, the following solution is obtained.

$M_1 = 28.1$	$T_1 = .0333$	$S_1 = 63.3$
$M_2 = 35.0$	$T_2 = .0403$	$S_2 = 88.3$
$\beta_1 = .8577$	$n_1 = 513$	$D = 1006.3A$
$\beta_2 = .7980$	$n_2 = 551$	

If B is increased by 15% to 709,971, then

$M_1 = 32.4$	$T_1 = .0384$	$S_1 = 73.0$
$M_2 = 41.4$	$T_2 = .0476$	$S_2 = 104.5$
$\beta_1 = .8696$	$n_1 = 506$	$D = 1275.5A$
$\beta_2 = .8155$	$n_2 = 540$	

Note that the increase in M_1, T_1, S_1 is 15.3% and for grade 2 the increase is 18.3% which results in a 26.8% increase in achievement.

Sisson explains that his models may not be valid but are illistrations of the types of analysis and models that can be applied to education. Concerning the difference in programs, Sisson notes that by using aggregate behavior, such differences are not as important and are, in fact, results of undetermined characteristics of the programs. Additionally, the difficulty in estimating the parameters is mentioned.

Of the models reviewed in this paper, this model has utilized the operations research approach more than any other. The development and discussion of the model are quite clear and well illustrated. Sisson mentioned that the model will be tested using data from the Philadelphia school system, but did not describe how the constants in the model were obtained. One major deficiency in the analysis is that expenditure for different kinds of materials, staff, space is not allowed. For example, instead of just using teachers in a school, what if a mixture of teachers, paraprofessionals and guidance counselors costing the same amount were used. Because paraprofessionals are much less expensive than teachers or guidance counselors, the amount of staff per student would increase, by a_i, but the achievement level, D_i, may not correspond to that level resulting from an increase in teachers of a_i since paraprofessionals may cause achievement to happen at a different rate. This deficiency can be removed by adding another staff term for each type of staff involved in instruction.

Similarly, additional terms for materials and space can be added. One very important question is how detailed should the model be? When can staff types or materials or space be aggregated? By testing the sensitivity of each input variable to total cost and then ranking each variable by its relative effect, one can select an arbitrary effect level below which variables can be aggregated.

As mentioned previously, the selection of various variables as representative of their respective areas can be challenged. A more detailed description of how the particular

variables were chosen, supported by theory or observation from other educational programs, would enhance the paper's quality. In spite of the above comments, the work by Sisson is of much better quality than that being done by educational researchers and is quite good for the preliminary stages of modeling the educational process.

Sisson's second model utilizes a set of difference equations to predict student enrollment. Only the most general set of conditions will be discussed below. This non-linear theoretical model was included because it predicts the flow of students through a school system based on the outcome of the previous year.

Define:

α as the percentage of students not promoted to the next grade,

β as the percentage of students that are promoted to the next grade.

If K is the number of students admitted each year to the first grade and is a function of time, and α, β are not constants, then the grade enrollment n_i are also a function of time. Assume that K increases linearily, then

$$K(T) = K(T - 1) + \pi(\text{constant})$$

which implies

$$n_i(t) = (1 + \alpha_i)K - \alpha_i \pi$$

$$n_i(t) = \beta_{i-1} - n_{i-1}(t - 1) + \alpha_i n_i(t - 1) + O_i$$

where O_i is the measure of other factors in grade i.

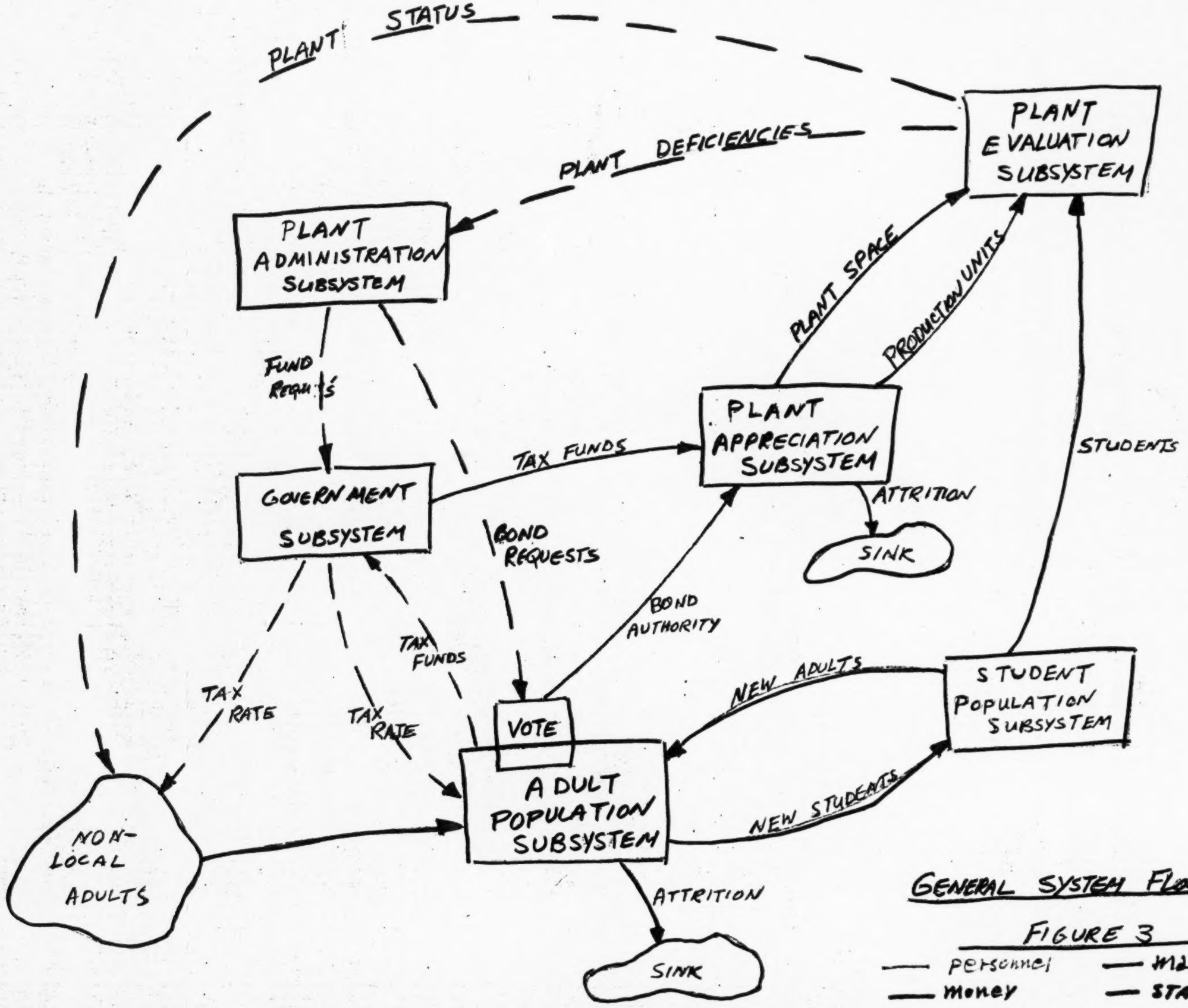
An example describing how a school administrator could use both models in conjunction to plan the teacher-student ratio for the next three years is given.

Several comments can be made about the validity of this model; however, one must remember that to build a model without a supporting theory is extremely difficult.

The assumptions made about student flow are subject to question and not very general, but this is one of the few models of an educational system where the present status is used to predict future effects. Quite important are these feedback effects for they convert the static model to one that utilizes some past information.

Because a simulation model proposed by T. H. Jones (13) incorporates several feedback loops, further comment on Sisson's model will be reserved for later. Jones' paper approaches the modeling of the public school system from a governmental-community point of view, not just the local administrator's view. To cover the additional areas of government and community action, a lesser amount of detail was included in the model. See Figure 2 for the General System Flow.

From the hypothesized flow, Jones constructs a Dynamo program including several feedback loops with delays to simulate the public school system. One gross oversimplification was that a production unit replaces all teachers, books, busses, custodians, and other items. Estimation of requirements for space was based on 50 square feet per student. Several other parameters such as construction delay and plant life were estimated from unknown origin.



GENERAL SYSTEM FLOW

FIGURE 3

In addition to estimating the needs of the school system, the model also predicts tax rates, bond approval, and immigration into the community. Unfortunately Jones' model is not easily reproducable because definitions for 59 variables or constants are necessary.

Even with the numerous variables the validity of the model is subject to question. This model's usefulness is not in predicting the actual needs of a school system, but is the demonstration that the interaction and delay effects can be modeled and, even on a gross level, yield some interesting insights into the relationships. Criticism of the values of the variables or the initial conditions are neglected here in order to consider the more severe deficiencies in the model.

One major problem is that two different programs cannot be compared using Jones' model. There exists a slim possibility that if the two programs could be transformed into common units of production that a comparison could be made; however, such units of production may not exist, let alone be readily computed. Hence optimum allocation of resources is not possible with this model. Such a deficiency removes from consideration one of the primary purposes of educational modeling. By testing several sets of values for the parameters, some insight can be gained as to the general effect of some of the variables; however, many sets of data would be necessary. Because the model has many feedback loops, the effects of certain changes may not be easily discernable. Methods of alleviating some of these difficulties will be presented in the author's model below. Several of the simplifying assumptions are unrealistic

such as no school dropouts are permitted, the actions that the government takes are immediate, and income level has no effect on voting, housing values or birth rates. This model was unusual for two reasons: (1) It considered feedback effects and time delays; (2) It examined the school system from a total point of view not just a school administrator's view.

The five papers reviewed are representative of the types of work currently being done in educational cost effectiveness. Up to this time, correlational studies have been the overwhelming majority of reports. By presenting the other types of models, the author has shown how these alternate models can be used and their limitations. The criticism of the models hopefully will lead to improved models and possible application of techniques from other fields. As the above models indicate, cost effectiveness in education is in a very primitive state when compared to cost effectiveness in military or industrial applications. One possible source of ideas relating to education cost effectiveness is the area of manpower training. The military and the poverty agencies conduct many programs some of which have been evaluated by the cost effectiveness approach. Private industry also trains people. Some or all of these program evaluations may be applicable to elementary and secondary education. More effort is necessary to search for better ways of doing educational cost effectiveness.

A Resource-Achievement-Utility Model

The author proposes as further research in educational cost effectiveness that a model derived from several models discussed above be developed. For the purpose of optimally allocating resources to yield the greatest utility, a procedure which combines cost incurrance, prediction of outcomes and conversion of those outcomes into units of utility is necessary. Interaction effects need to be considered. A combination of Sisson's, Amor and Dyer's, and Jones' models seems an appropriate starting point. Using Jones' general model but implementing several more detailed sub-models or modules to improve the accuracy of the proposed model, the new model would be able to perform two functions: (1) budgeting, (2) cost effectiveness. Budgeting will not be discussed, but it is a straight-forward application of the more detailed model.

By developing the proposed model, three things can be accomplished. First resource allocation between programs can be improved to yield greater utility. Second, allocation of resources within a program also can be improved. Lastly, some rudimentary relationships between specific inputs or programs can be related to total utility. This can aid in the design of new programs or into the allocation of resources.

Following are flow charts of the author's model. Starting with the student flow model proposed by Sisson, without the restriction that the entering population has a linear increase in size, computations for each time period are

completed. The particular time period chosen is the length of one quarter, semester, or school year, depending on the purpose of the model. Because the model is a simulation-type model, the restrictions made on various parts of the model to facilitate finding an analytical solution are relaxed.

One interesting possibility is that if the relationships in the model are deterministic or can be described adequately by a deterministic relationship, the model becomes a recursive set of functions. This eliminates some of the difficulties associated with a simulation model, such as how many times should the simulation be done to insure accurate results.

To present the model some definitions are necessary:

Program - a particular activity in the school system such as reading, arithmetic, etc.

Facility - an object that depreciates because of time and not usage and has a lifetime greater than one year.

Supply - an object with a lifetime of less than one year.

Staff - persons that are directly involved in the performance of a program.

Direct - type of item or idea relating to a type of instruction.

Indirect - type of item or idea relating to support of instruction, but is not actual instruction.

Usage related object - lifetime depends only on the number of times object is used.

Achievement re- an object that is related only to the
lated object - amount of achievement

Structure of the Model

Only a conceptual model will be presented because of the lack of reliable information describing the relationship of the resources and other variables. Some specific equations will be suggested for parts of the model. One major advantage of this model is its modularity; that is, several of the components can be reproduced as necessary.

The calculations start with the Student Flow modules. For each sub-population, a student flow module is constructed. The number of entering students is recorded. If failure rates and dropout rates are independent of the programs used, the rest of the Student Flow module calculations are done. Otherwise the next step is a decision module. One example of the equations for student flow not dependent on the program used is

$$G_i = g_i S_i$$

$$S_i = (\alpha_{i-1})S_{i-1} + E_i$$

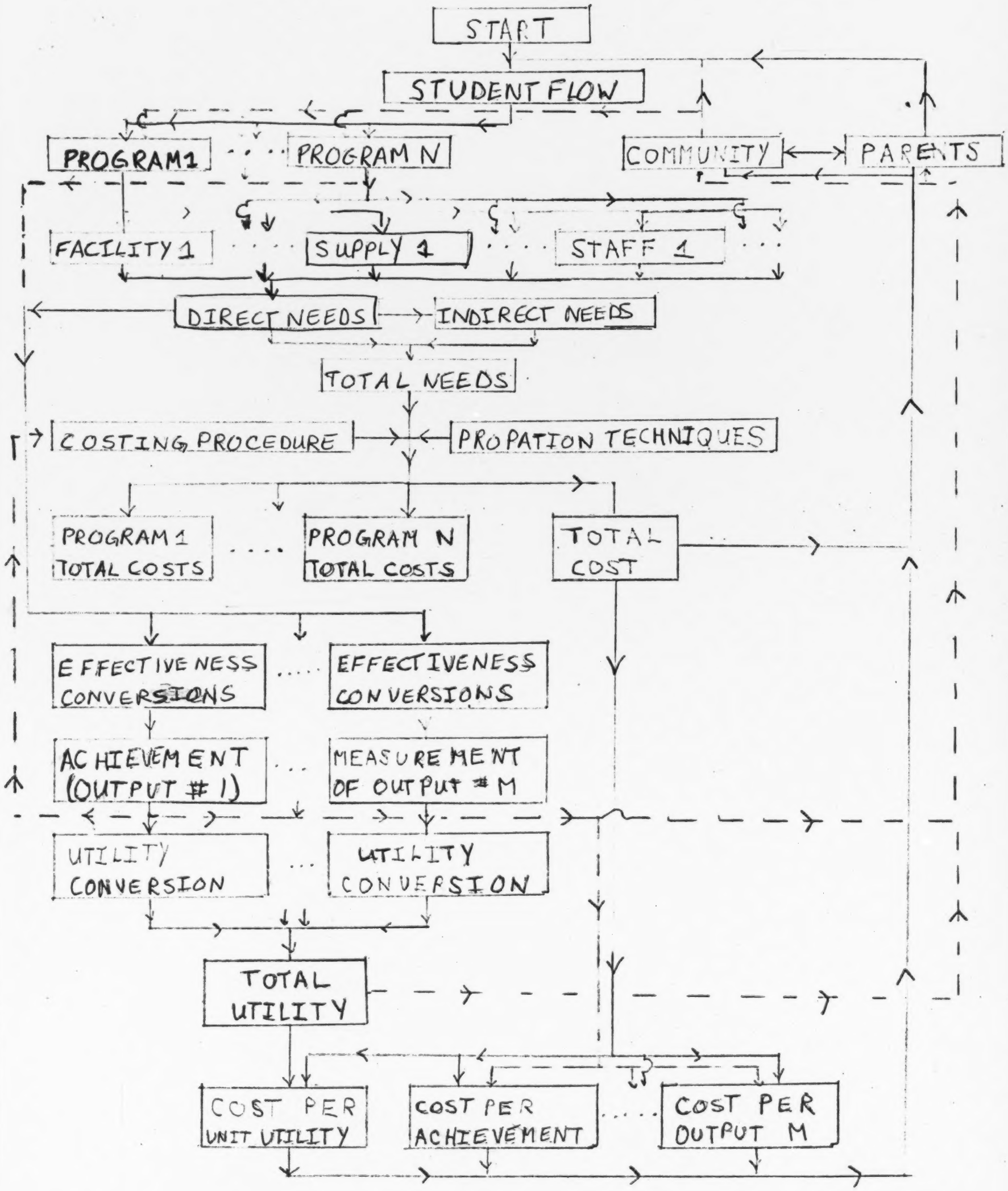
$$D_i = d_i S_i$$

$$g_i + \alpha_i + d_i = 1$$

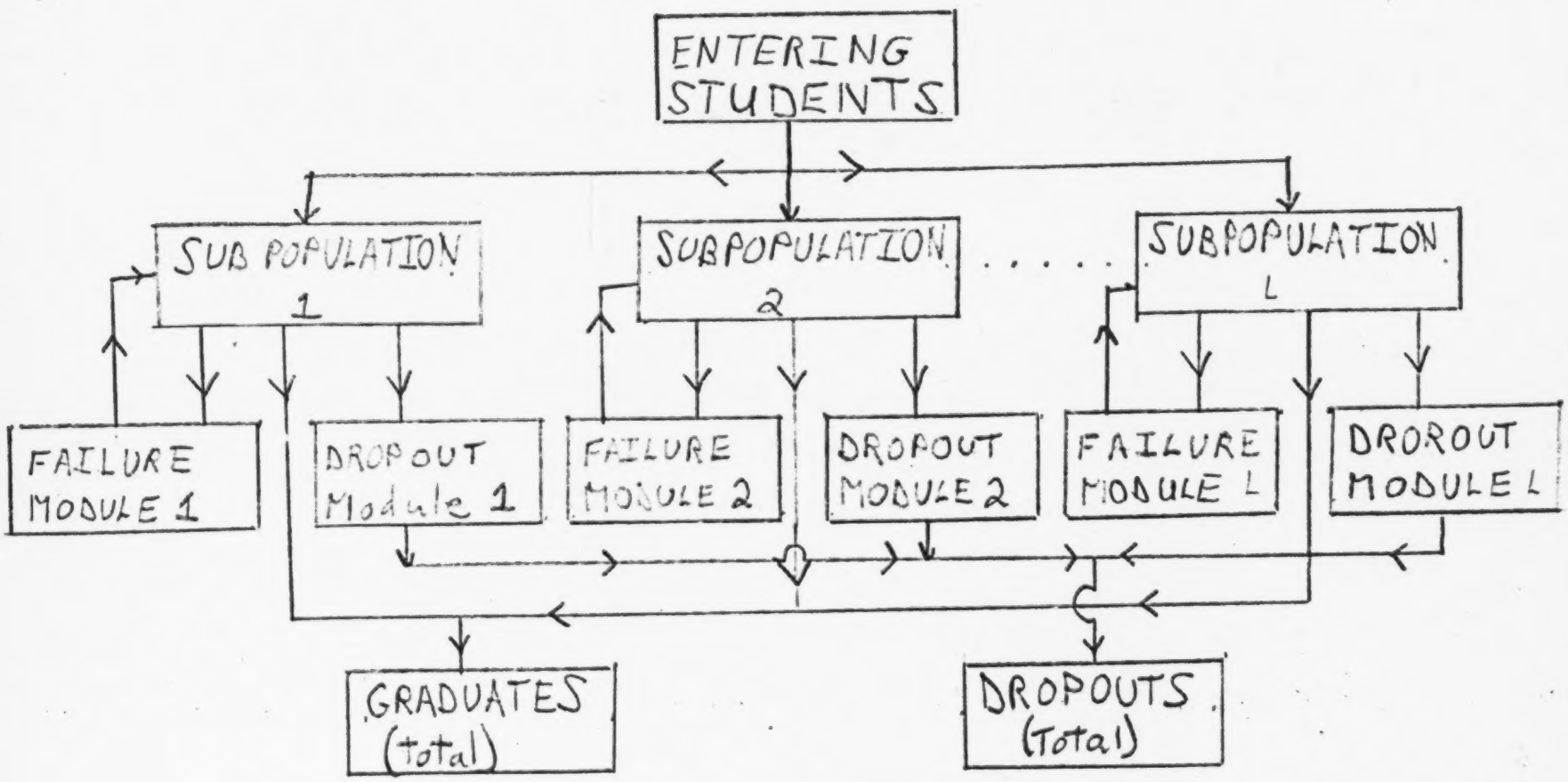
S_i is the number of students in the ith time period,

α_{i-1} is the proportion of student from the i-1 time period continuing in the population,

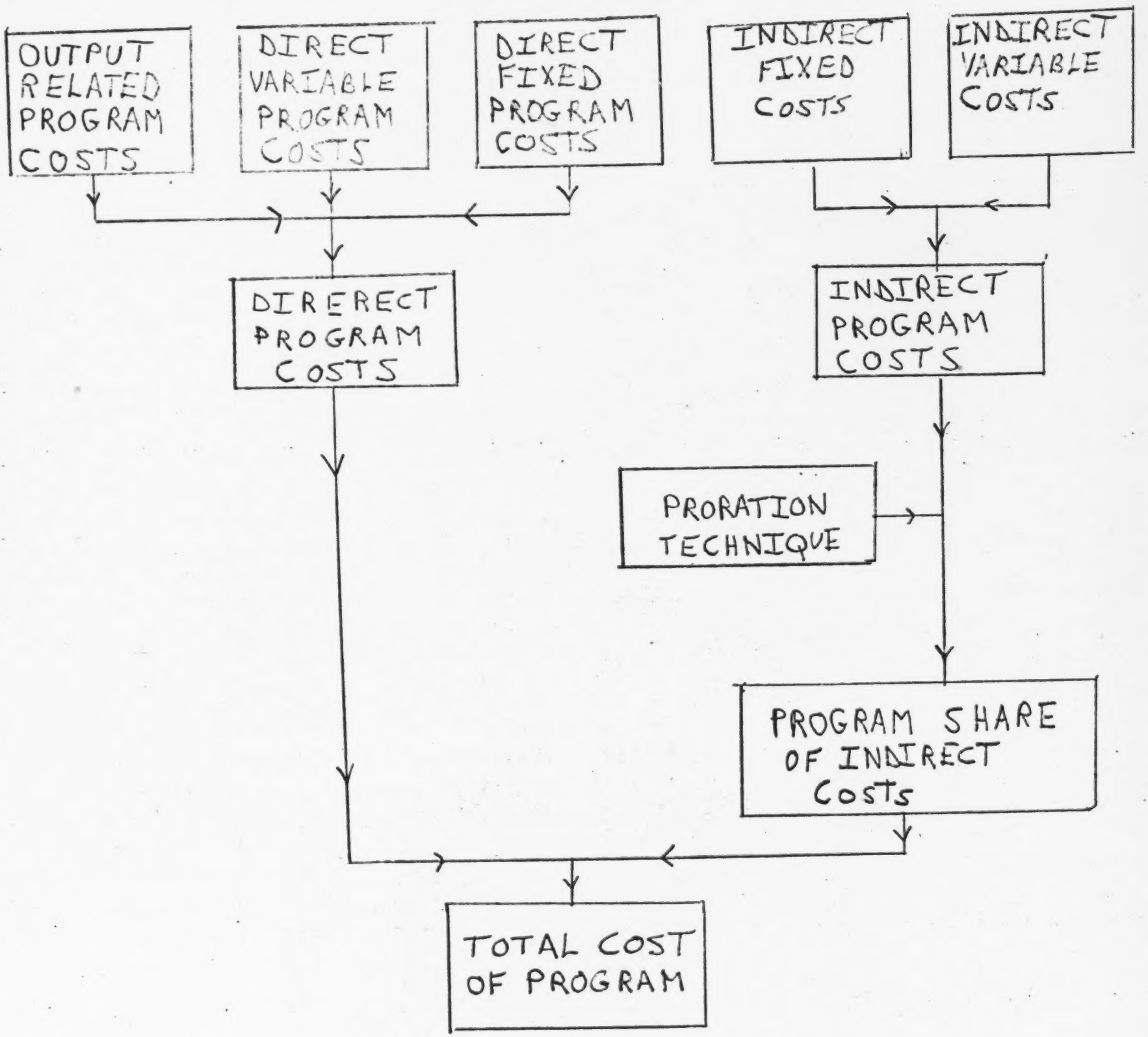
E_i is the number of student entering the population at



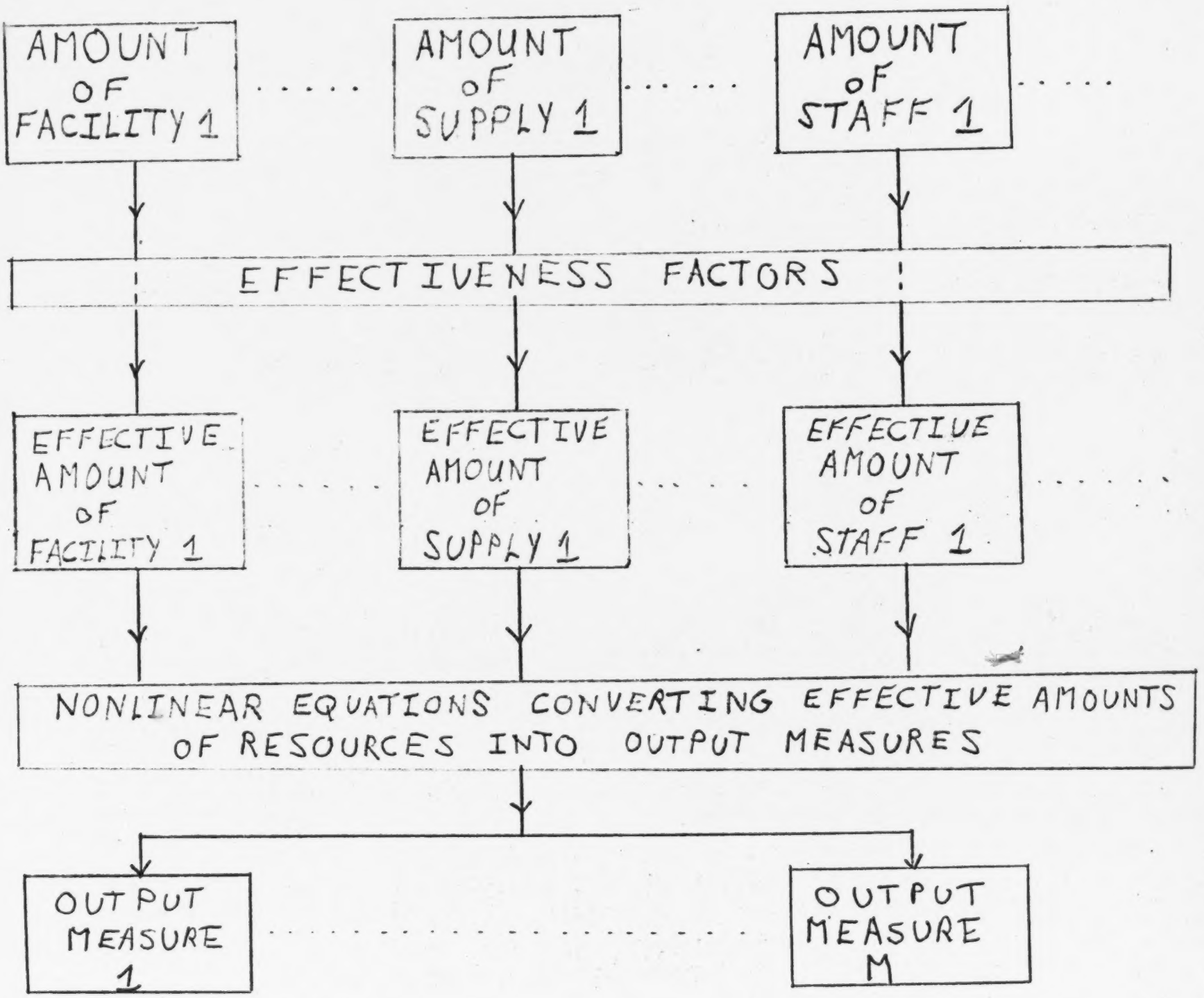
MODULE STRUCTURE



STUDENT FLOW



TOTAL COST OF A PROGRAM


AMOUNT OF FACILITY 1
AMOUNT OF SUPPLY 1
AMOUNT OF STAFF 1
EFFECTIVENESS FACTORS
EFFECTIVE AMOUNT OF FACILITY 1
EFFECTIVE AMOUNT OF SUPPLY 1
EFFECTIVE AMOUNT OF STAFF 1
NONLINEAR EQUATIONS CONVERTING EFFECTIVE AMOUNTS OF RESOURCES INTO OUTPUT MEASURES
OUTPUT MEASURE 1
OUTPUT MEASURE M

EFFECTIVENESS MODULE

time i,

g_i is the graduation rate at time i,

d_i is the dropout rate at time i,

G_i is the number of graduating students in period i,

D_i is the number of dropouts in period i.

If the equations for student flow depend on the programs selected then the equations might be similar to those of Sisson's Student Flow model.

Choosing a decision procedure for the allocation of resources depends on the nature of the equations in the model and the constraints. If the equations and constraints are convex differentiable, then the optimal allocation of resources can be found by setting the marginal utility per dollar of each program equal to that of the other programs. Another approach is to transform the system of equations into another system of equations that can be solved by integer programming or some non-linear technique. Additionally, an iterative numerical technique could be used to determine which programs to include. Further comments on this vexing problem will be made during the discussion of the sensitivity analysis.

After determining the programs to be funded, each student sub-population generates a need for resources such as staff time, facilities, and supplies. Need is based not only on the student sub-population and programs, but also on a scheduling pattern. For example if the maximum number of students in a classroom was X then the classroom must be large enough to accomodate X students simultaneously regardless of the total

number of students that use the classroom during the day. These needs for resources will vary so widely among programs that no specific equation representing the resource needs can be presented. A general form could be

$$A_{ik} = \sum_j f_i(S_j, s)$$

where A_{ik} is the amount of resource k needed for program i,

S_j is the number of students in sub-population j,

s is a scheduling factor,

f_i is a function that converts the number of students into a resource requirement.

After the direct program needs have been determined, the indirect needs are determined. These needs are based on the number of students in each sub-population and on the particular programs considered.

From indirect and direct needs, the total need for resources is computed. This total need is modified by a schedule-usage pattern to yield total adjusted needs. For example, if a reading program generates a need for one classroom hour and an English program generates a need for five classroom hours, then one classroom used six hours could fill this need.

A possibly non-linear relationship converts the need for resources into dollar costs. If this model is used for budgeting purposes, one preliminary step is necessary--total available resources is subtracted from total resource need to yield net resource needs for the current time period. One example of the costing procedure is the COST-ED model (10) which has

a detailed linear costing procedure. The COST-ED model was designed to determine costs by program and to show the importance of specific variables to total cost. An integral part of the costing procedure is the use of proration techniques. Few of the resources in a school system are used to support just one program. The proportion of costs attributed to a program by the proration techniques is then very important. One difficult question is how to prorate fixed costs. The approach favored by the author is to determine a cost per student hour and charge fixed costs to a program based on the number of student hours in that program. Other methods besides time can be used as cost proration techniques. Some items, such as slides, depreciate based on usage and not time. In that case a usage related proration technique would be better. Additionally, some expenditures could be based on the amount of achievement attained. Proration by units of achievement would be appropriate. The total cost of a program is determined by summing all the prorated costs.

Next the effectiveness is computed. No generally acceptable equation exists. Sisson's first model provides one such equation. The author proposes the following equation:

$$0 = \sum_i A_i n_i \prod_k (\xi_{ik} + S_{ik})\left(\frac{t_k}{Ti}\right)(E_{ik})^{\gamma_{ik}} \prod_k (\psi_{ik} + F_{ik})\left(\frac{t_k}{Ti}\right)(E_{ik})^{V_{ik}} \prod_k (\phi_{ik} + M_{ik})\left(\frac{t_k}{Ti}\right)(E_{ik})^{N_{ik}} \cdot \quad f(C,P,I,R)$$

where A_i, ξ_{ik}, μ_{ik}, Ψ_{ik}, V_{ik}, ϕ_{ik}, N_{ik} are constants,

O is output measured by some device,

n_i is the number of students in program i,

S_{ik} is the ratio of staff type k to students in program i,

t_k is the time a particular resource k is used in program i,

T_i is the total time in program i,

E_{ik} is the effectiveness multiplier of the kth resource in program i,

F_{ik} is the amount of facility k per student in program i,

M_{ik} is the amount of materials and supplies of type k per student in program i,

f(C,P,I,R) is a function of community and parents, innate student characteristics and rewards.

Of course this equation can be reporduced for each output. If student sub-populations have different values for the variables in the above equations, the right hand side can be modified by substituting two subscripts, i and j, for the subscript i. The subject j would denote which sub-populations was involved. Summation of the right hand side would be over subscripts i and j.

Because the output of each sub-population can be determined, a method of assigning utility, such as Amor and Dyer's model, can be used. Total utility can be determined by summing

the individual output utilities.

The community, in which the author includes government, acts on the parents and students directly. Additionally, the community acts directly and through influences on parents on both direct and indirect programs. Lastly, the parents affect the community, student flow and achievement modules.

A set of equations based on the outputs of this time period is solved to determine the effects on the parents and community n time periods later where n is the information delay time. Also there will be another feedback effect, which depends on the outputs, on the effectiveness equations. An example of this type of equation is a reward or incentive bonus based on production. One possible reason why minority groups do not do as well in school as white Americans is that the reward to minority students who have increased education is small. This would be similar to having a production line with small or no incentives to increase production.

A summarization module, the, computes cost per unit of achievement for each outcome and cost per unit of utility. This cycle is repeated for each time period and cumulative results are determined.

One interesting analysis which should be done is the sensitivity of the cost per unit achievement and cost per unit of utility to each variable. By ranking these variables, the most important contributors can be identified. This identification indicates which variables should have highest priority in future research. Once the variables that have the most effect are understood, then the variables contributing

lesser amounts can be analyzed.

Using this method focuses attention on those variables that are the most important. After each revision in the model another sensitivity analysis can be done indicating a new set of important variables. Furthermore, those variables that are least important can be aggregated without much loss in analytical power of the model.

The sensitivity analysis can also be used to determine solutions with greater total utility. If the optimal solution to the non-linear equations of the model cannot be found and an initial solution is known, the sensitivity analysis can suggest which programs can be removed and which should be added to the initial solution. Using the criterion of greatest marginal utility per dollar may yield a better solution, but not necessarily the optimal one. As with the other methods of solving non-linear equations, the specific equations will determine which approach is best.

Conclusion

This model was constructed to stimulate interest in this field and hopefully yield some insight into areas that need more work. Determination of the parameters and their values is one obvious step that must be done. Until this time, the cost effectiveness models examined only a small part of the total picture of the effects of the school system. Elementary though this model may be, the author hopes that it will produce some insight or stimulate thinking in the area of educational cost effectiveness.

This author has reviewed five papers on education cost effectiveness that represent the different methods currently being used. As indicated, several areas besides public education may have techniques that can be applied to educational cost effectiveness. Suggestions were made on how to improve each of the models represented. Obviously this is a starting point for further research.

As one possible improvement the author presented a general model that considers the relatiionship of specific inputs to the overall educational system. Of special importance was the acknowledgement of feedback effects with time delay. Certainly the determination of the delay times and effect should be investigated. Lastly, the author presented the sensitivity analysis as a method of determining the priority of investigating variables.

Using this model and the expected improvements, the author hopes that educators will conduct experiements to determine the validity of this approach with the possibility of determining the parameters and relationships between the variables.

FOOTNOTES

1 See Lyle (20), p. 143.

2 See "A Survey of School Effectiveness Studies" by James Guthrie in Do Teachers Make a Difference?, p. 35.

3 See Levin (18), p. 57.

4 Ibid, pp. 57-58

5 Ibid, p. 63.

6 See Sisson (27), p. 113.

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